An investigation of adiabatic spiral vortex flow in wide annular gaps by visualisation and digital analysis

C. C. Wan* and J. E. R. Coney*

This paper, the third in a series describing experimental investigations into spiral vortex flow, presents visual evidence illustrating adiabatic transition modes in a wide gap of radius ratio 0.848. Also, power spectra, relating to velocity fluctuations, are found to be comparable for two working fluids, oil and air. Good agreement has been found between these results and those published by the authors in which diabatic transition modes were related to the heat transfer characteristics of the flow at various axial Reynolds numbers

Key words: *turbulent flow, flow transitions, vortices, visualisation*

A viscous fluid contained in an annular gap, formed by a stationary outer and a rotating inner cylinder, undergoes successive instabilities as the speed of the latter is increased; this phenomenon has been the subject of numerous theoretical and experimental investigations. This problem is further complicated by the addition of a pressure gradient in the axial direction, giving the familiar spiral form of the Taylor instability. The authors have published their experimental findings for adiabatic[1] and diabatic[2] flow.

Among the early contributions to this study, Donnelly and Fultz[3] and Snyder[4] found good agreement between their experimental results and Chandrasekar's[5] theoretical work. Observing the flow patterns at $Re_a = 40$, Snyder noted that the toroidal cells acquired a spiral form which persisted until $Re_a = 150$. Above $Re_a = 160$, the cells were seen to develop sinusoidal waves along their axes, which increased in number with Re_a and Ta. Schwarz *et al*[6], in their experiments, found that these azimuthal waves were formed from the interaction of the toroidal and spiral modes. For axisymmetric flow, excellent agreement was found to exist between their results and the theory of Krueger and Di Prima[7].

With the imposed axial flow increased to $Re_a = 200$, the flow becomes more complex and attempts to study the flow transitions by visual means have not been wholly successful. The introduction of hot wire anemometry techniques, however, has made investigations into these higher ranges possible. Kaye and Elgar[8], using both hot wire anemometry and visual techniques, found, as they varied Re_a and Ta, four distinct flow regimes: laminar, laminar plus vortices, turbulent, and turbulent plus vortices. By observing the hot wire signals on an oscilloscope, the initial onset of vortices was detected by the laminar trace, a straight line, suddenly changing to a regular sinusoidal form. The breakdown of this to an irregular aperiodic trace marked the transition from the laminar plus vortices to the turbulent regime. For the transition from the turbulent to the turbulent plus vortices regime, they stated that: 'it was felt that this boundary line was not as clearly measurable or discernible as the other three boundaries'. In addition, they were aware of the effect of geometric effects on Taylor instability and modified the Taylor number by a geometric constant. Astill[9], considering developing spiral flow, also observed these four flow regimes.

Other experimental workers, notably Flower *et al*[10], Gravas and Martin[11], Coney and Simmers[12] and Sorour and Coney[13], confirmed the transition from the laminar to the laminar plus vortices regime, but encountered difficulty in determining the other transitions. In consequence, it has been questioned if such accurate demarcations are possible.

Recent developments in digital signal processing, however, have led to an improvement in the understanding of these complex secondary flow transitions. In 1973, Gibson[14] developed digital techniques for the recording and processing of laminar and turbulent flow data. He showed that the digital computer can perform complicated analogue operations with comparable accuracy. More recently, a qualitative study of Taylor vortex flow instability was carried out by Fenstermacher *et al*[15], using laser doppler and digital techniques. Using velocity spectra, they observed that the transitions from periodic vortex flow to quasiperiodic, asymmetric and finally to chaotic flow gave distinctive power spectra.

It is the object of the present investigation to obtain, for a wide annular gap, an insight into these

* Department of Mechanical Engineering, University of Leeds, Leeds, UK, LS2 9JT
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transitions by means of visual techniques, flow visualisation methods and inner cylinder torque measurements. It is intended that these data should provide a confirmation of the transformations reported earlier[1].

Apparatus

Two sets of apparatus were used. Apparatus A, for flow visualisation, was of radius ratio 0.848 and used oil as a working fluid; apparatus B, of radius ratio 0.8, used air.

Apparatus A

The apparatus, described elsewhere[16] (Fig 1), consisted essentially of two cylinders, the eccentricity of which was variable but which, in the present tests, were concentric. The outer cylinder, of inner surface diameter 114.3 mm, was made of 'Perspex' and was stationary. The inner cylinder, of diameter 97.0 mm, was of steel and could be rotated. The working length of the annular gap so formed was 1067 mm; the working fluid was Shell Tellus oil R5 with aluminium paint pigment as the flow visualisation medium. An imposed axial flow through the annular gap was provided by a centrifugal pump; the mass rate of flow of oil was determined by means of orifice plates, the pressure difference across them being measured by a water manometer. The temperature of the oil was measured by thermocouples inserted at three positions along the annular gap, the temperature of the oil being controlled by an immersion heater in the oil storage tank. The rotational speed of the inner cylinder was determined by a magnetic pick-up, activated by a toothed wheel mounted on the inner cylinder shaft; the number of pulses was displayed on a digital counter. The inner cylinder torque was measured by an inductive torque transducer, and was indicated on a digital torque meter.

Fluctuations in the flow velocity were measured by means of a DISA 55R11 hot film probe placed at mid-gap. Having a nickel film deposited on a 70 μm diameter quartz fibre, the overall length was 3 mm with a sensitive film length of 1.25 mm. The film was protected by a 2 μm quartz coating. By use of the immersion heater and because of the large volume of oil in the circuit, it was possible to obtain a virtually constant temperature during an experimental run, thus eliminating a source of error in the probe output.

The availability of a VAX 11/780 on-line computer, together with an analogue to digital converter (adc), gave an accurate and rapid means for recording and analysing analogue signals. Signals from the digital inner cylinder speed counter, the torque meter and the anemometer were sampled at a specified sample interval, Δt, by the adc; these signals were converted to point voltages and stored in the computer.

In the investigation of the inner cylinder torque-speed relationship, two channels of the adc were used, both channels being simultaneously sampled.

Signal analysis for power spectra and auto-correlation were achieved on the computer. At a sample interval, Δt, of 2 ms (for a maximum frequency range of 250 Hz) and with a mobile graphics terminal beside the apparatus, rapid analysis of power spectra and auto-correlation functions were possible. For longer time records, however, this method was not convenient due to the considerable computational time involved. Sample record lengths consisted of 33 seconds of recording time, giving 16 384 samples. The data was stored in a data file, with the files resulting from several days' investigation being stored on magnetic tape. Complete documentation of the main computer program and the general arrangements of the analogue and digital equipment is given elsewhere[1].

Apparatus B

This apparatus and its attendant instrumentation has also been described in detail elsewhere[1]. Used in its wide gap configuration, it comprised a stationary outer cylinder of diameter 139.7 mm and a rotatable inner cylinder of diameter 111.8 mm; the axial length of the annular gap thus formed was 1820 mm. The outer cylinder was of mild steel and the inner was of Tufnol. The working fluid was air, which flowed down the annular gap. Measurements were made at 1482 mm from the inlet, which was well within the region of fully developed flow.

Results and discussion

Power spectrum and auto-correlation results

The evolution of the flow in apparatus A, with Shell Tellus R5 oil as the working fluid, for $N = 0.85$ and $Re_a = 250$, is shown in Fig 2. These studies in the wide gap reveal a more complicated flow than that

Notation

adc	Analogue to digital converter
b	Annular gap width $(R_2 - R_1)$
d_e	Equivalent diameter of annulus $(2b)$
f	Frequency
N	Annular radius ratio (R_1/R_2)
$P_x(f)$	Power spectral density function of signal $x(t)$
R_1	Radius of inner cylinder
R_2	Radius of outer cylinder
$R_x(\tau)$	Autocorrelation function of signal $x(t)$
Re_a	Axial Reynolds number $U_m d_e/\nu$
t	Time
T	Inner cylinder torque
Ta	Taylor number $2\Omega_1^2 R_1^2 b^3/(R_1+R_2)\nu^2$ for a wide gap
Ta^*	Taylor number ratio (Ta/Ta_c)
Ta_c	Critical Taylor number
U_m	Mean axial velocity component
Δt	Sample interval
μ	Dynamic viscosity of fluid
ν	Kinematic viscosity of fluid (μ/ρ)
ρ	Density of fluid
τ	Time delay of correlation function
Ω_1	Inner cylinder angular velocity

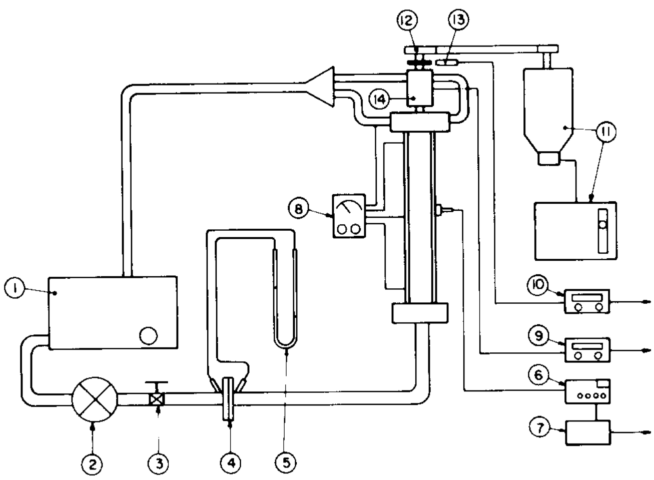


Fig 1 Schematic arrangement of apparatus A: 1–oil storage tank; 2–pump; 3–valve; 4–orifice plate; 5–manometer; 6–DISA anemometer unit; 7–DISA low pass filter; 8–Comark thermometer; 9–torque meter; 10–digital counter; 11–variable speed drive; 12–drive belt; 13–magnetic pick-up; 14–torque transducer. Outputs from 7, 9, and 10 (lower right) to Vax 11/780 computer via adc

prevailing in the narrow gap[1] and correspond well to the results of the wide gap investigations reported in that paper[1].

Considering the power spectra, Fig 2, the spectrum at the critical Taylor number, $Ta_c = 14\,700$, is very complex and no sharply defined peak is discernible. It is also continuous with a broad spectral peak B, which is periodic. With further increase of inner cylinder speed, a chaotic spectrum evolves (Fig 2(b) and (c)) but with broad spectral peaks being absent. However, after $Ta \approx 250\,000$ (Fig 2(d)), sharp spectral peaks appear out of the continuum, which broaden and become established with increase of Ta, (Fig 2(e) and (f)).

The auto-correlograms, corresponding to these spectra, are also given in Fig 2, the periodic nature of the flow being observed in Fig 2(a). A loss of correlation is observed with the emergence of the chaotic mode, together with a short decay time (Fig 2(b) and (c)). The reversion of the auto-correlograms to a periodic function (Fig 2(d), (e) and (f)) corresponds to the advent of the broad spectral peaks. The periodicity of the function still persists at the maximum obtainable Ta, 1.66×10^6. The visual detection of these flow transitions is extremely difficult, owing to the gradual appearance of the higher frequency modes. However, the auto-correlograms, in retaining their periodicity, provide a useful means of determining the flow characteristics.

The spectra and auto-correlograms taken, with air as the working fluid, from apparatus B for $N = 0.8$ and $Re_a = 500$, are shown in Fig 3. These results have a remarkable similarity to those in Fig 2.

Visual results

The developments of spiral vortex flow at $Re_a = 250$ can be seen from Fig 4. At the critical Taylor number

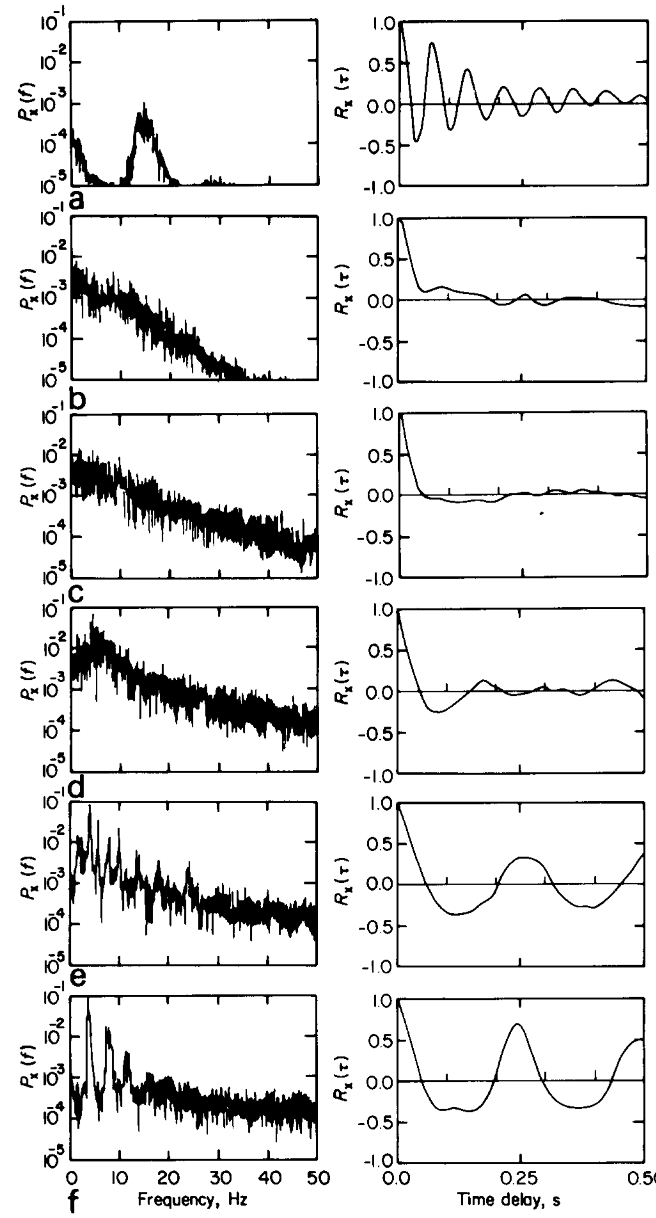


Fig 2 Power spectra and auto-correlograms for oil, $Re_a = 250$, $N = 0.848$: *(a)* $Ta = 14\,700$ *(b)* $Ta = 23\,400$ *(c)* $Ta = 66\,800$ *(d)* $Ta = 247\,000$ *(e)* $Ta = 1.02 \times 10^6$ *(f)* $Ta = 1.66 \times 10^6$

($Ta_c = 14\,700$), the spiral form of the Taylor instability with distinct cell patterns is observed moving along the annular gap (Fig 4(a)). An increase in the Taylor number to 20 700 is seen to cause the breakdown of the spiral wave form, which spreads from the outlet to the inlet of the annular gap; the presence of waviness and randomness in the cell patterns is evident (Fig 4(b)). At $Ta = 28\,000$, chaotic flow has appeared (Fig 4(c)), apparently resulting from the superimposition of many waves on each other. It is of interest that this chaotic wavy mode spirals in the opposite sense to the previous modes. This pattern persists until $Ta = 260\,000$, when a cellular structure reappears (Fig 4(d)). However, these cells have the orientation of those of Fig 4(a) and (b). Further increase in the Taylor number causes strengthening of this pattern with the pairs of cells now being seen as contra-rotating (Fig 4(e)). The spectral results of

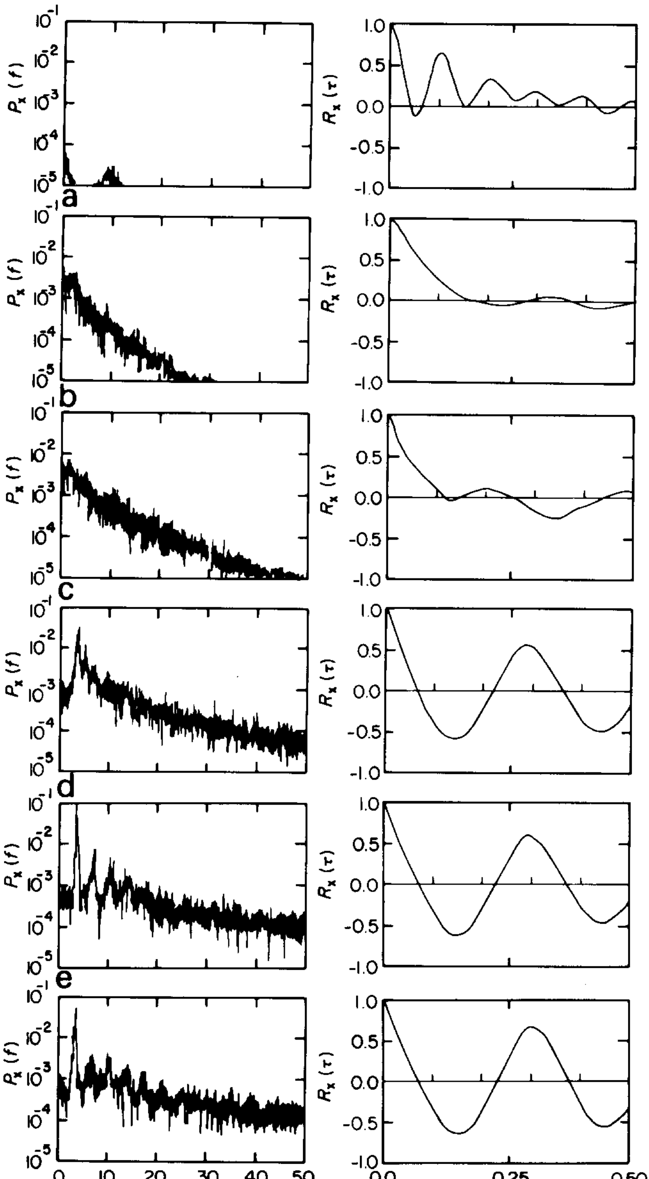


Fig 3 (above) Power spectra and auto-correlograms for air, Re$_a$ = *500,* N = *0.8: (a)* Ta = *11 000 (b)* Ta = *90 000 (c)* Ta = *400 000 (d)* Ta = *1.7×10⁶*

Figs 2 and 3 suggest that this mode is periodic, and vigorous, and similar to the toroidal forms occurring at the critical Taylor number with zero axial flow. It can be observed, also, that a small scale motion resembling turbulence is present within each cell (Fig 4(e)). It was found that, as the axial flow increases, so the difference in the Taylor number at which the spiral vortices and the chaotic wavy mode occur increases. In addition, the axial length of each spiral vortex cell increases with decreasing axial flow.

Comparison of spectral and visual results

The results of inner cylinder torque measurements taken from apparatus A with an axial Reynolds number, Re_a, of 250, are shown in Fig 5. These measurements indicate a constant torque—Taylor number relationship after the onset of the spiral vortex mode. For $20\,000 \leqslant Ta \leqslant 40\,000$, there is a gradual decrease in the torque, this being reflected in the visual observations as the breakdown of the spiral vortices occurs and the chaotic wavy mode is established (Fig 4(b) and (c)). The power spectra taken under these conditions (Fig 2(b) and (c)) confirm the chaotic nature of the flow. The higher transitional modes are seen to cause a monotonically increasing inner cylinder torque—Taylor number characteristic, with no indication of any abrupt changes occurring.

Comparison with other investigations

O'Brien[17], studying experimentally the vortex patterns at Taylor numbers in the region of 10^8, discovered the presence of spiral vortices wrapped around the basic vortex cell. He inferred that highly oriented large eddy structures could form part of the turbulence at these high Taylor numbers. Recent investigations of Barcilon *et al*[18] and Koschmieder[19] into supercritical Taylor vortex flow substantiate this finding. Barcilon *et al*, considering the range $10^3 \leqslant Ta^* \leqslant 10^5$, reported a transition at high Taylor num-

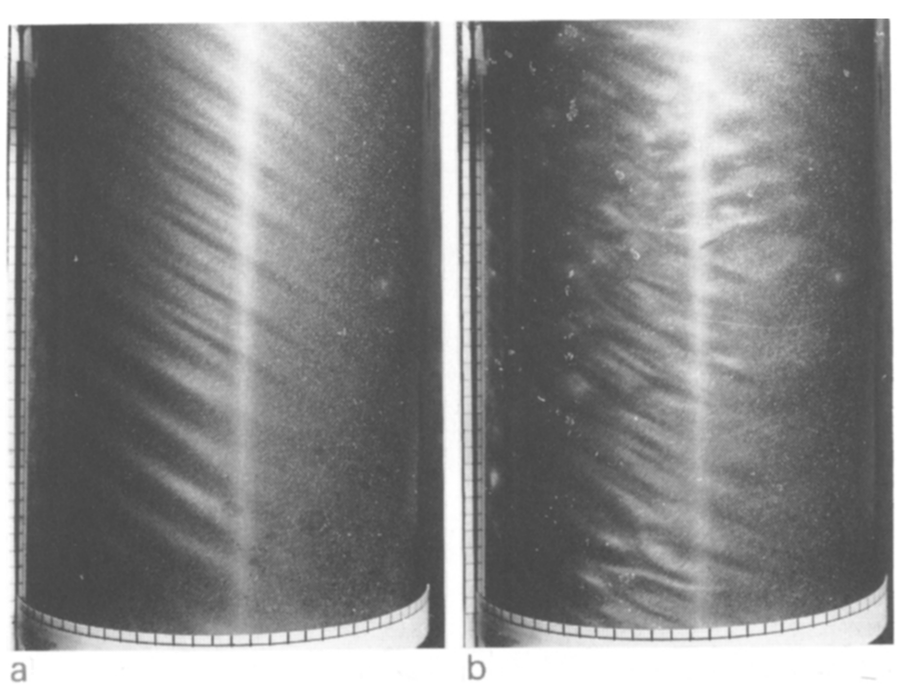


Fig 4 (right) Visual results for oil, Re$_a$ = *250,* N = *0.848: (a)* Ta = *14 700 (b)* Ta = *20 700 (c)* Ta = *28 000 (d)* Ta = *260 000 (e)* Ta = *1.05×10⁶* bers from a complex flow pattern to a more ordered disturbance, having a low wave number at the cell core. They suggested that the flow in the gap was on two distinct scales, ie the scale of the gap width which was filled with turbulent vortices and the much smaller scale of the boundary layer which contained Görtler vortices, observed at the outer wall as a herring-bone pattern; it is possible that these latter were identical to the spiral vortices noted by Ozogan[16]. This evidence agrees well with that presented in Fig 4(e). Koschmieder measured the wavelength of the vortices up to $Ta^* = 40 \times 10^3$ for $N = 0.72$. His results indicated that, at very high speeds, the flow in the annular gap took the form of toroidal turbulent vortices of uniform size. He also found that the wavelength of these turbulent vortices was substantially greater than the wavelength of the initial Taylor vortices occurring after the breakdown of Couette flow.

From the evidence so far discussed, the sharp spectral peaks observed at the higher Taylor numbers in Figs 2 and 3 seem to be due to the formation of such turbulent vortices. It should be pointed out that, while the studies discussed above were at zero axial flow, if Re_a is sufficiently small, the axial flow component is not significant compared with the rotational component and flow transitions will be similar to those observed for $Re_a = 0$. The decrease in inner torque with Taylor number for $20 \times 10^3 \leq Ta \leq 40 \times 10^3$ is an interesting phenomenon. In a recent paper by the authors[2] on diabatic spiral vortex flow in a wide annular gap, the plot of Nusselt number related to the outer surface of the gap versus Taylor number shows a similar decrease in Nusselt number after the formation of spiral vortices. The occurrence is over a similar Taylor number range and the coincidence is remarkable. At the time the paper was written, no conclusive explanation of this decrease could be found but, in the light of the present results, it may be attributed to the breakdown of spiral vortex flow into the chaotic mode. Also in that paper, it was reported that the vigorous nature of the chaotic flow enhances the convective heat transfer at the outer stationary annular surface and a third flow regime, consisting of turbulent vortices, causes a decrease in Nu with increase in Taylor number for Re_a. A reflection of this effect is not evident in the present torque measurements but it is suggested that there is a possibility of detecting it by taking measurements of outer cylinder torque or shear stress.

Conclusions

1. With the increase of axial Reynolds number, the transition to the chaotic mode occurs at values of the Taylor number which progressively

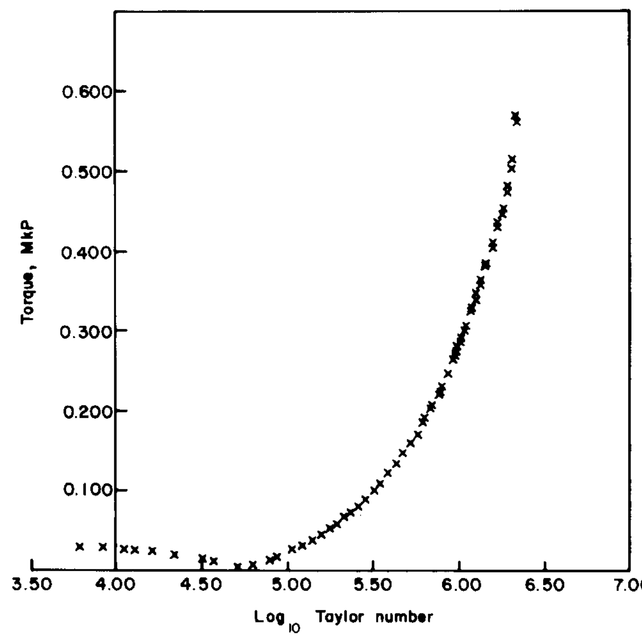
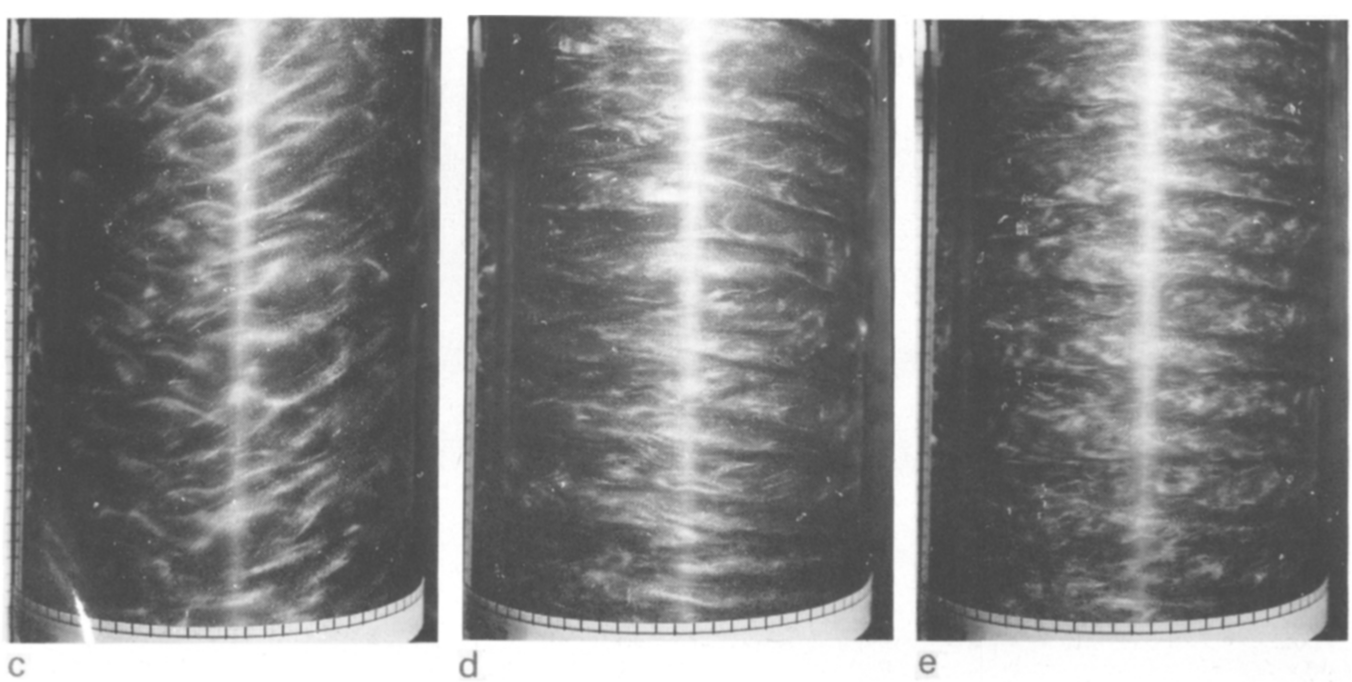


Fig 5 Variation of inner cylinder torque with Taylor number for oil, $Re_a = 250$, $N = 0.848$



approach the critical Taylor number for the initial onset of vortex flow.

2. Visual observations, using photographic techniques, have confirmed results from power spectra, auto-correlograms and inner cylinder torque measurements.

3. Good confirmation of the transition modes investigated previously[1,2] has been made.

4. The final transition to be observed in the present study is for $Ta = 2 \times 10^6$, where the flow is reorganised into a more stable and periodic turbulent vortex mode.

5. The breakdown of the spiral vortices into the chaotic wavy mode is accompanied by a decrease in the inner cylinder torque.

6. Both air and oil were found to be acceptable working fluids for signal analysis.

7. At high Taylor numbers for the present range of axial flows, the axial Reynolds number loses its significance, in that similar transitions to those for the zero axial flow condition were observed.

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